

# **Assessment of Courses in Mathematics and Statistics for General Education Category 4: Mathematical/Logical Reasoning**

## **Methodology**

The following courses in the Department of Mathematics and Statistics were assessed during the Fall Semester of 2001-2002: Math 110, 112, 113, 115, 121, 180, 201 and Stat 154. The assessment was conducted by Dr. Ernest Boyd and Mr. Robert Bohland reviewing the final examinations in these classes of all students whose last digit of their student identification numbers was seven, i.e., approximately ten percent of all students registered for the courses. There were 164 students in this group. The total number of test problems reviewed was 2922, i.e., on average 17.82 problems per student.

In order to assess the first objective/outcome for Category Four, each problem was classified as follows. A problem was labeled as an historical application of a mathematical system if it was a more traditional topic common to similar courses in past generations, such as rational functions, basic trigonometric functions, arithmetic sequences, mean and variance of simple data sets. A problem was labeled as a contemporary application of a mathematical system if it was considered to have more recent emphasis, such as logarithmic functions, exponential functions, inverse trigonometric functions, geometric sequences, matrices, and statistics with probability distributions. Each category was further divided into problems that required the student to identify the mathematical system employed versus problems that required the student to describe the mathematical system. For example, a problem where the mathematical abstraction was given would be considered identification, while a word problem requiring the student to develop the mathematical symbolism would be considered description.

To assess the student's ability to apply mathematical reasoning to simple problem-solving strategies, higher-order problem-solving strategies and modeling strategies, the following categories were used: (1) problems requiring symbolic manipulation for essentially one step to derive the solution, (2) problems requiring symbolic manipulation for multiple steps to derive the solution, (3) problems requiring a modeling approach to derive the mathematical system needed for the solution, and (4) problems requiring a higher-order problem-solving strategy where the results of one procedure are used as input to another procedure needed to derive the solution. Each problem was then evaluated on two scales. First, the student's solution was scored on a percentage to measure the quality of the presentation in written mathematically narrative form. Consideration was given to determine if the solution was well organized and ordered logically. Second, the student's solution was scored on a percentage to determine if it constituted a valid mathematical argument, i.e., it described the parts of a valid proof for that problem.

## **Results**

On average a student in one of these classes was asked to solve approximately 18 problems of which 64.2% required the student to be able to identify an historical application of a mathematical system., 19.3% required the student to be able to identify a contemporary application of a mathematical system, 7.6% required the student to be able to describe some component of an historical application of a mathematical system and 9.5% required the student to be able to describe some component of a contemporary application of a mathematical system.

Ninety-three percent of all problems required students to present their solutions in a written mathematically narrative form. The following percentages assess the quality of the written presentations for an average problem within this set. The average student was 76.4% successful in presenting a written mathematically narrative form for a problem requiring symbolic manipulation for essentially one step to derive the solution. The average student was 66.8% successful in presenting a written mathematically narrative form for a problem requiring symbolic manipulation for multiple steps to derive the solution. The average student was 70% successful in presenting a written mathematically narrative form for a problem requiring a modeling approach to derive the mathematical system needed for the solution. The average student was 52.7% successful in presenting a written mathematically narrative form for a problem requiring a higher-order problem-solving strategy where the results of one procedure are used as input to another procedure needed to derive the solution.

All problems were evaluated for the validity of the mathematical argument or proof. The following percentages assess the ability of the average student to explain a valid solution of an average problem within each category. The average student was 72.7% successful in presenting a valid solution for a problem requiring symbolic manipulation for essentially one step to derive the solution. The average student was 63.3% successful in presenting a valid solution for a problem requiring symbolic manipulation for multiple steps to derive the solution. The average student was 63.2% successful in presenting a valid solution for a problem requiring a modeling approach to derive the mathematical system needed for the solution. The average student was 54.5% successful in presenting a valid solution for a problem requiring a higher-order problem-solving strategy where the results of one procedure are used as input to another procedure needed to derive the solution.

## **Conclusions**

Considering the results of the survey we can conclude that on average a student is satisfying the first objective/outcome in Category Four. On average a student was required to identify and describe both historical and contemporary applications of mathematical systems. However, the percentage of problems that require the students to describe the mathematical abstraction of the problem, rather than merely identifying the symbolism given, needs to be increased.

For the second and third objectives/outcomes in Category Four the results of the survey indicate some mastery of expressing mathematical ideas in writing as well as explaining a valid mathematical argument. On average for all problems in all categories the average student was 68.5% successful in presenting a written mathematically narrative form and 65.5% successful in presenting a valid solution. However, there is room for improvement in the future. A realistic goal would be to raise all percentages to a minimum of 70%.

The fourth objective/outcome in Category Four indicates the weakest point in the average student's performance. Certainly applying higher-order problem-solving strategies is more difficult since it requires a more holistic and less mechanical thought process. But it should be the ultimate goal for any educational endeavor. There is need for significant improvement here. There needs to be more emphasis placed in this area.

## **Recommendations for Changing the Rubrics for Category Four**

1. Eliminate the distinction between historical and contemporary applications. Mathematics and logic are universal in both space and time. There is no value in distinguishing between problems this way since all mathematical and logical applications have merit today and in the future.
2. Classify applications of mathematical and logical systems based on the degree of complexity and abstraction employed to solve the problem. We recommend four categories:
  - (1) Problems where the student is given the mathematical/logical symbolism and needs to identify the appropriate problem-solving strategy or proof. These problems would need essentially one step to derive the solution.
  - (2) More complex problems where the student is given the mathematical/logical symbolism and needs to identify the appropriate problem-solving strategy or proof. These problems would require symbolic manipulation for multiple steps to derive the solution.
  - (3) Problems requiring a modeling approach to derive the mathematical/logical system needed for the solution. These problems would require the student to describe the mathematical or logical abstraction in order to set up the symbolism used to derive the solution.
  - (4) Problems requiring a higher-order problem-solving strategy where the results of one procedure are used as input to another procedure needed to derive the solution. These problems require the students to develop logical connections between different components of the subject.
3. Combine the first and fourth objectives/outcomes into the following three parts:
  - (1) Identify the mathematical/logical abstraction and symbolism needed for a given application of a mathematical/logical system.
  - (2) Describe the mathematical/logical abstraction and symbolism needed to model a given system.
  - (3) Apply higher-order problem-solving strategies connecting separate elements in a mathematical/logical system.

**First objective/outcome: Illustrate historical and contemporary applications of mathematical/logical systems.**

In order to assess the first objective/outcome in mathematics and statistics, the following chart indicates the percentage of the problems in each category as interpreted in the appendix.

	Historical	Contemporary
Identify	64.2%	19.3%
Describe	7.6%	9.5%

In order to assess the first objective/outcome in philosophy, the following chart indicates the number of students within certain ranges of proficiency in identifying and/or describing applications of logical systems.

Rate of Mastery	Rate of Mastery	Rate of Mastery
Below 70%	Between 70% and 85%	Above 85%
1 Student	5 Students	6 Students

From these results we conclude that the average student's knowledge of applications of mathematical/logical systems has increased, but we need to increase the percentage of problems requiring students to describe the application of the mathematical/logical system as apposed to merely identifying the application of the mathematical/logical system.

**Second objective/outcome: Clearly express mathematical/logical ideas in writing.**

In order to assess the second objective/outcome in mathematics and statistics, the following chart indicates the level of proficiency for the average student in presenting a well written narrative form to answer an average question in that subset.

Subset 1	Subset 2	Subset 3	Subset 4
76.4%	66.8%	70%	52.7%

In order to assess the second objective/outcome in philosophy, the following chart indicates the number of students within certain ranges of proficiency in presenting a well written narrative form.

Rate of Mastery Below 70%	Rate of Mastery Between 70% and 85%	Rate of Mastery Above 85%
1 Student	4 Students	7 Students

From these results we conclude that while the average student's ability to present a well written narrative form has increased, students need to improve their ability to organize a written presentation of a problem requiring a higher level of mathematical/logical abstraction involving multiple steps and higher-order problem solving strategies.

**Third objective/outcome: Explain what constitutes a valid mathematical/logical argument (proof).**

In order to assess the third objective/outcome in mathematics and statistics, the following chart indicates the level of proficiency for the average student in presenting a valid mathematical proof of the solution to an average problem in the subset.

Subset 1	Subset 2	Subset 3	Subset 4
72.7%	63.3%	63.2%	54.5%

In order to assess the third objective/outcome in philosophy, the following chart indicates the number of students within certain ranges of proficiency in presenting a logical argument.

Rate of Mastery Below 70%	Rate of Mastery Between 70% and 85%	Rate of Mastery Above 85%
1 Student	4 Students	7 Students

From these results we conclude that while the average student's ability to present a valid mathematical/logical argument has increased, students need to improve their ability to solve problems requiring a higher level of mathematical/logical abstraction.

**Fourth objective/outcome: Apply higher-order problem-solving and/or modeling strategies.**

In order to assess the fourth objective/outcome in mathematics and statistics, we compare the rates of success for problems in Subset 4 to the rates of success in all problems. The following chart indicates the level of proficiency for the average student in mastering an average problem in Subset 4 as compared to an average problem for all subsets.

	Subset 4	All Problems
Written Form	52.7%	68.5%
Valid Solution	54.5%	65.5%

In order to assess the fourth objective/outcome in philosophy, the following chart indicates the number of students within certain ranges of proficiency in applying higher-order problem-solving strategies.

Rate of Mastery	Rate of Mastery	Rate of Mastery
Below 70%	Between 70% and 85%	Above 85%
6 Students	4 Students	2 Students

In conclusion this objective/outcome indicates the weakest point in the average student's performance. Certainly applying higher-order problem-solving strategies is more difficult since it requires a more holistic and less mechanical thought process. But it should be the ultimate goal for any educational endeavor. There is need for significant improvement here. There needs to be more emphasis placed in this area.

## Assessment of Courses in Philosophy General Education Category 4: Mathematical/Logical Reasoning

The work of nine students from three different sections of Philosophy 110, Introduction to Logic and Critical Thinking was assessed using the four assessment rubrics for category 4 courses. Also, the work of four students from Philosophy 112, Logic and Scientific Method, was assessed in a similar fashion. Our assessment consisted of examining three different tests taken by each student. Test 1 examined students on matters dealing with standard distinctions between terms and statements, including the denotation/connotation distinction, assessments of truth-status of statements, and basic distinctions between propositions, including causal, hypothetical, analogical, etc. Test 2 examined student understanding of informal fallacies, different modes of language use (informative, evaluative, directive, expressive), and basic logical forms (categorical syllogisms, hypothetical syllogisms, statistical syllogism, Venn diagrams, etc.). Test 3 tested students' ability to assess statements and arguments for basic logical properties like consistency, logical truth and validity, by using formal techniques of proof (i.e., propositional calculus).

### **Results**

Each of the 4 rubrics contains at least two, and as many as four, levels of comprehension. Our assessment began with identifying which of the three tests, and what parts of each test, were most relevant to which of the 4 rubrics. (See discussion section for more on this matter). This was not easy to determine and our choices are not, we believe, the only way of connecting tests with rubrics and vice-versa. Following this, we assessed the level of mastery of each student by using a system utilizing a +, 0, and – system. The ‘+’ was used to designate superior mastery of the particular assessment rubric, the ‘0’ for adequate mastery, and the ‘-’ in cases where the rubric was not adequately mastered. All told, there were 12 different levels of mastery within the four separate rubrics to assess. An appropriate symbol was given for each of these 12 levels. Results are recorded and analyzed on a separate sheet.

### **Discussion of Assessment Procedure and Results**

The single most difficult problem with our assessment was determining what components of the exams to hook up with which parts of the assessment rubric. A very close second most difficult problem was deciding what counted as mastery of particular rubrics. For example, rubric 1 speaks of identifying and describing applications of math/logical systems. One question raised here is what to count as satisfying or not satisfying this rubric on the student's part. If a single case of identification or description was taken to constitute mastery (i.e., taking a yes/no interpretation of the rubric) then the level of mastery would, it seems, be quite high. For it is difficult for a student to take three exams in a critical thinking course and to not have demonstrated ability to identify or describe an application of a logical system on at least one occasion. Simply understanding some the exercises on the exams requires, we would contend, the ability to identify and describe logical systems, and logical principles. As such, only students who failed the course could be said to not have demonstrated an ability to identify or describe a logical system. We decided to interpret the rubrics so as to allow for the possibility of grading levels of mastery, although it was difficult to render such an assessment in precise terms. We looked over the various exams and made assessments of levels of mastery in the particular rubrics in much the same way that one does so generally in grading exams. We basically considered things like how well and how often students were able to identify, describe, explain, etc., logical concepts and principles.

We are not certain that our approach is in line with the original intentions of those who created the rubrics. As such, we are not certain that our approach is going to answer the sorts of questions to which those who requested a general education assessment are trying to find answers.

Ultimately, I think our approach could be regarded as something of a compromise, a *via media*, between these two different courses of interpretation. We did not, for example, see any way in which we could connect each problem on each exam with particular rubrics. However, some portions of the particular tests were more readily connected with particular rubrics and in such cases, the question of a student's satisfying a rubric or not was largely a matter of how well s/he performed on that portion of the exam. However, even in such cases, the determination of a student's level of success at meeting a particular rubric was not determined simply by echoing the original test score which a student received on that portion of the exam. And that is because the particular questions on the exam in these various sections were often designed to test skills and understanding that was not identical to the skills and abilities tied to particular rubrics.

More particularly, it became clear to us that the tests we examined were designed to test a student's ability to *do* logic and critical thinking rather than to test student's abilities to talk about what they were doing. But many of the rubrics seemed designed to determine the extent to which students had or had not required a certain kind of meta-level competence, as opposed to the ability to perform various logical tasks.

We assumed, for the most part however, that one's proficiency (or lack thereof) at the logical exercises on the exams was the best gauge of the student's ability to identify, explain or describe the logical concepts and ideas contained in the respective rubrics. To assume otherwise, we felt, would render our assessment rather trivial. For as noted above, it was clear to us that every test-taker could be said on at least one occasion or another in the exams to have successfully identified, explained or described the basic logical notions mentioned in the rubrics. In some sense, simply being able to take the exams at all is evidence of an ability to identify, describe or explain basic logical concepts. Thus, we felt it best to tie (at least in part) a student's success or failure at meeting the various rubrics to his/her level of performance on the particular exams.

We believe, however, that a clarification of the rubrics is in order. In particular, the point of the rubrics needs to be made clearer. What are the rubrics supposed to be doing? Are they, e.g., supposed to provide instructors with a guide for course content and course exams? That is, are they to serve as guides to instructors as to what their courses should be trying to do, what their students should be able to do, at the end of their courses? Or are we to regard the rubrics as already fitting our courses, in some sense, so that assessment is and should be simply a matter of determining how many of our students do or do not satisfy the rubrics, or how well they do so?

New Assessment Rubric #1: (1) Demonstrate ability to apply a math/logical system; (2) Demonstrate ability to apply more than one math/logical system

New Assessment Rubric #2: Explain or illustrate understanding of a mathematical/logical idea in written narrative form; (2) Explain or illustrate understanding of a mathematical/logical idea in written narrative form.

New Assessment Rubric #3: (1) Demonstrate ability to do or identify valid proofs; (2) Demonstrate ability to distinguish valid proofs from invalid proofs; (3) Demonstrate ability to understand or explain the properties/characteristics of valid proofs and the properties/characteristics of invalid proofs

As for rubric #4, it would be nice if there were some explanation of how we are to understand the distinction between "simple problem solving" and "higher order problem solving".