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Minnesota State University, Mankato HOLD and CLEAR buttons only compatible with Acrobat V. 4 and 5
Curriculum Proposal

Please type or select the requested information. Print completed forms, add appropriate paper attachments, and route through MSU's curricular process for recommendations and decisions.

(Check all that apply):		Proposal # 212				
College: <u>Science, Engineering and Technology</u>	<input checked="" type="checkbox"/> Undergraduate	Effective Date of Change:				
Department: <u>Mathematics and Statistics</u>	<input type="checkbox"/> Graduate	Academic Year 05-06				
Program: _____	CIP # _____	(For Office Use Only)				
Type of Change: <u>COURSE PROPOSALS</u>		<table border="1" style="width:100%; border-collapse: collapse;"> <tr> <th>Course Designator and Number</th> <th>Number of Credits</th> </tr> <tr> <td>Math 497 492</td> <td align="center">3</td> </tr> </table>	Course Designator and Number	Number of Credits	Math 497 492	3
Course Designator and Number	Number of Credits					
Math 497 492	3					
Proposed: <u>N/A</u>		(if applicable)				
Title Current: _____						
Title Proposed: <u>Mathematics Capstone Experience</u>						
24-Char. Abbrev: _____						

Include a course or program description for the Bulletin (30-40 words maximum for courses, 100 for programs):

Description This course is designed to allow undergraduate students an opportunity to integrate their undergraduate mathematics experiences by engaging each student in working on a problem in applied or theoretical mathematics. In doing so, students will see connections between the various topics found in the undergraduate mathematics curriculum. Content will vary by semester. An important component of this course will be the preparation and presentation of a research paper describing the student's progress toward a solution of the problem under consideration. Problems will arise from the course content and materials as presented by the instructor. Because of the breadth of mathematical topics needed for successful completion of the course, students need to have senior standing.

Prerequisites: Two of the following: Math 316 – Intermediate Analysis, Math 345 – Abstract Algebra, or Math 375 – Introduction to Discrete Mathematics and senior standing (or permission of the instructor). Course also can be taken as an independent study with permission of a cooperating faculty member. F, S

Rationale or Justification for change:

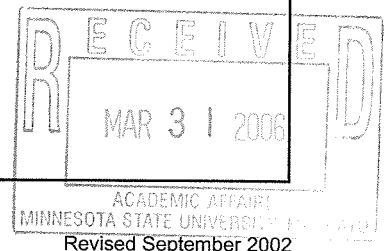
See attached

*****For General Education or Cultural Diversity Courses Only*****

<p align="center">General Education Course:</p> <table border="1" style="width:100%; border-collapse: collapse;"> <thead> <tr> <th align="left">GE Category #</th> <th align="left">GE Category Name (Maximum of 3 Categories)</th> </tr> </thead> <tbody> <tr><td align="center">N/A</td><td> </td></tr> <tr><td align="center">N/A</td><td> </td></tr> <tr><td align="center">N/A</td><td> </td></tr> </tbody> </table> <p><small>? For Writing Intensive Courses, attach a description of the kind and quantity of writing. ? For Upper Division Courses, include a description of the respects in which it is broad and general rather than narrow and specific, and so suitable as GE.</small></p> <p>Attach paper copies of the following:</p> <ol style="list-style-type: none"> Syllabus or course outline. Course's student learning outcomes associated with each GE competency or CD designation. List of strategies to be used to assess students' achievement of each GE competency or CD designation. 	GE Category #	GE Category Name (Maximum of 3 Categories)	N/A		N/A		N/A		<p align="center">Cultural Diversity Course:</p> <p align="center"><small>(Please check one.)</small></p> <p><input type="checkbox"/> Core (At least 75% devoted to topics of race, gender, sexual orientation, age, class, and disabilities as they occur in United States Society.)</p> <p><input type="checkbox"/> Related (At least 25% devoted to the above topics or to a global perspective on topics related to African American, Asian, Hispanic, and Native American inhabitants of the United States.)</p>
GE Category #	GE Category Name (Maximum of 3 Categories)								
N/A									
N/A									
N/A									

*****For New Courses*****

(Check all that apply): <input checked="" type="checkbox"/> Course is an elective. <input type="checkbox"/> Course is required for program <input type="checkbox"/> Pre- or Co-requisites: <input type="checkbox"/> Other courses are being changed or eliminated. (Explain.) _____	Instructional Type: <u>Lecture</u> Grading Format: <input checked="" type="checkbox"/> Grade <input type="checkbox"/> P/N <u>Math 316,345, 375 (at least two of these)</u>	Course will be offered: <input checked="" type="checkbox"/> Fall Semester <input checked="" type="checkbox"/> Spring Semester <input type="checkbox"/> Summer Session
<input type="checkbox"/> Course content or title is similar to courses in other departments. (Attach copy of letter of agreement with other program(s) contacted. Indicate the nature of the discussions and/or resolution of differences or potential conflicts.)		
Attach paper copies of the following: <ol style="list-style-type: none"> Syllabus or course outline. Course's student learning outcomes. A list of resources required to offer and support this course. A description of how teaching this course will affect department staffing. If 400/500 level course, an explanation of added expectations of graduate students. 		





Minnesota State University, Mankato
Curriculum Proposal

*****For Program Proposals*****

Attach paper copies of the following:

- a. Student learning outcomes for the program.
- b. Minutes from department and college curriculum meetings in which action was taken on this proposal.
- c. Program Assessment Plan. Forms are available on the Academic Affairs Web site:
http://www.mnsu.edu/acadaf/words/PRA_SampSLOAssessPlan.doc
- d. List of program requirements for **New** programs, or a list of **Current** and **Proposed** program requirements for **Redesigned** programs.
- e. A list of resources required to offer and support this program.
- f. A description of how offering this program will affect department staffing.
- g. A list of additional library holdings required for this program.

Please include rationale for any proposed changes in number of program credits:

*****For Programs Requiring MnSCU Approval*****

If any of the following changes are proposed, please fill out and attach MnSCU Program Approval Forms, which are available on the Academic Affairs Web site:

<http://www.mnsu.edu/acadaf/html/currformsprocesses.htm>

1. **Creation** of an entirely new program.
2. **Redesign** of existing programs, which takes any of the following forms:
 - ? Addition or deletion of a program option. Options are part of program design in which 30-50% of the courses are required as part of a common core for all students, and which offers curriculum alternatives greater than 30% of the total number of credits in the major. Options are appropriate to baccalaureate or masters programs.
 - ? Addition or deletion of a program emphasis. Emphases are part of program design in which more than 50% of the courses are required as part of a common core for all students, and which offers curriculum alternatives with a minimum of nine credits. Emphases are appropriate to associate and baccalaureate programs.
 - ? Change in program name.
 - ? Change in program CIP #.
 - ? Change in TOTAL program credits.
 - ? Change in degree award. For example, changing a B.A. to B.S.
 - ? Creation of a new degree award in a related academic area. Examples include creation of a certificate program from an existing degree program, or a new degree program from an existing degree program (e.g., Art History BA from Art BA.)
3. **Relocation** of an existing program. This is a proposal to move an existing program from one site to be exclusively offered at another site, and requires closing the program offered at the original site. For example, a program offered both on-campus and through extended campus is to be offered only at the extended campus site.
4. **Replication** of an existing program. This is a proposal to offer an existing program at a new site, which may be an existing MnSCU-approved site, or another campus of the same institution. Replicated programs are offered at both the original site and the new location.
5. **Suspension** or **reinstatement** of a program. This proposal suspends admission of students into an existing program, and is good for three years. Reinstatement proposals request the reopening of student admissions into a given program.
6. **Closure** of a program. This proposal requests closure of an existing program and its from an institution's official inventory of academic programs. Unless a department seeks to re-open a suspended program, it should be closed within three years of suspension.



Minnesota State University, Mankato
Curriculum Proposal

Signature Page

Department

Recommended (Category/ies _____)
 Not Recommended (Category/ies _____)

Larry Pearson 3/21/06
 Department Chair Date

Comments:

College Curriculum Committee

Recommended (Category/ies _____)
 Not Recommended (Category/ies _____)

Alan Mahoney 3-28-06
 Committee Chair Date

Comments:

College Dean

Recommended (Category/ies _____)
 Not Recommended (Category/ies _____)

[Signature] 3/28/06
 Dean Date

Comments:

General Education Subcommittee

Recommended (Category/ies _____)
 Not Recommended (Category/ies _____)

 General Education Subcommittee Chair Date

Comments:

Undergraduate Curriculum and Academic Policy Committee

Recommended (Category/ies _____)
 Not Recommended (Category/ies _____)

Mark Komize 4/28/06
 UCAP Faculty Chair Date

Comments:

Faculty Association Graduate Committee

Recommended
 Not Recommended

 Faculty Association Graduate Chair Date

Comments:

Graduate Dean

Recommended
 Not Recommended

 Graduate Dean Date

Comments:

Academic Affairs Council

Recommended (Category/ies _____)
 Not Recommended (Category/ies _____)

[Signature] 5/4/06
 Assistant Vice President Date

Comments:

Senior Vice President and Vice President for Academic Affairs

Approved (Category/ies _____)
 Not Approved (Category/ies _____)

[Signature] 5/4/06
 Sr. Vice President / Vice Pres. Academic Affairs Date

Comments:

Course proposal, rationale, syllabus, learning outcomes, department resources, and impact on staffing with regard to the proposed new course Math 497: Mathematics Capstone Experience (3 credits).

1. The proposal:

Description: This course is designed to allow undergraduate students an opportunity to integrate their undergraduate mathematics experiences by engaging each student in working on a problem in applied or theoretical mathematics. In doing so, students will see connections between the various topics found in the undergraduate mathematics curriculum. Content will vary by semester. An important component of this course will be the preparation and presentation of a research paper describing the student's progress toward a solution of the problem under consideration. Problems will arise from the course content and materials as presented by the instructor. Because of the breadth of mathematical topics needed for successful completion of the course, students need to have senior standing.

Prerequisites: Two of the following: Math 316 – Intermediate Analysis, Math 345 – Abstract Algebra, or Math 375 – Introduction to Discrete Mathematics and senior standing (or permission of the instructor). Course also can be taken as an independent study with permission of a cooperating faculty member. F, S

2. Rationale:

The proposed course is designed as a finishing school-type of course for our mathematics majors. Students will see connections between topics in mathematics, have an opportunity to read mathematics research, perform mathematics research at a modest level themselves, and practice communicating mathematics in both oral and written form. A course proposal and two sample syllabi for this course are attached. The course proposal includes a discussion of Instructional Goals, Learning Outcomes, Methods of Evaluation, and Department Resources.

The Department of Mathematics and Statistics designed this course to respond to Recommendation 5 of the Program Review of the Department of Mathematics and Statistics, academic year 2002-2003:

Recommendation 5

- Develop a capstone course or senior seminar or senior project with the following characteristics:
 - i. Connections are made between various topics found in core and non-core courses;
 - ii. A significant written project is required, preferably typeset using TeX or LaTeX;
 - iii. Oral presentations are made to Departmental faculty and students, perhaps fashioned after a music major's senior recital.

- Provide incentives for faculty to work with students
- Provide support for students to present their work at regional/national conferences

The Department committed to designing a Capstone Course in the Department Goals for 2003/4:

Goal 5. The Department of Mathematics and Statistics will enhance academic excellence in undergraduate studies by

(a) Investigating the feasibility of offering a Senior Capstone Experience;

The proposed course was discussed at the Department meeting on Friday, November 18, 2005 (minutes attached).

3. The proposed course is a topics course. Content will vary by semester and instructor. A generic course outline is provided which describes Goals, Learning Outcomes, Methods of Evaluation, and Staffing requirements. Two fully realized course syllabi are also attached (Dynamical systems and Enumerative Combinatorics). This course will not be available to graduate students.

Mathematics and Statistics Department
Meeting Minutes
Friday, November 18, 2005

Present: Boyd, Hannick, Hermann, Kitsul, Lee M.A., Lee N., Lee, W., Nelson, Olson, Pearson, Powers, Rahman, Rockswold, Singer, VanVeldhuizen, Waters, Wiest, Zuiker

Minutes of the last meeting approved by consensus.

Waters moved and Olson seconded, "We hire two mathematics education faculty this year. The motion passed.

Zuiker moved and Olson seconded, "Accept the prerequisite changes for 115 with the deletion of "or successful completion of Math 098 with permission of department chair" (*Must achieve a score of 19 or better on the MnSCU Math Readiness Test, or have achieved an ACT Math subscore of 20 or higher, or successful completion of MATH 098 with permission of the department chair. F, S*)

Olson moved and W. Lee seconded, the amendment "Go back to original statement of new prerequisites "

The Amendment passed.

The Motion passed.

Wiest moved and Olson seconded, "Accept prerequisites for Math 375" (*The current prerequisite for Math 375 is Math 180 or Math 290, we propose to change it to Math 247. This is proposed in response to 180 not being enough of a prerequisite for CS majors taking 375. They are taking 247 as part of their program, this will ensure they take it prior to 375 and will then have been exposed to proof prior to taking 375.*)


The Motion passed.

→ Discussed Capstone Course. Add offered F or S or one on one

Discussed TA's. Singer will offer an internship as in the TA report next semester.

Discussed volunteering for department committees.

The minutes were submitted by Dr. Mary Wiest.

 You replied on 3/20/2006 3:18 PM.

Attachments can contain viruses that may harm your computer. Attachments may not display correctly.

Singer, Dan W


From: Zuiker, Mark A

Sent: Mon 3/20/2006 2:59 PM

To: Singer, Dan W

Cc:

Subject: capstone

Attachments:  [Capstone Course Syllabus \(Revised\).doc\(79KB\)](#)

Hi,

Changes to the proposed course added since the last department meeting:

- 1) Allow the course to be offered both Fall and Spring as well as for independent study.
- 2) Remove the required Evaluation Criteria and make them suggested.

Dr. Mark A. Zuiker, Assistant Professor
Department of Mathematics and Statistics
Wissink Hall 273
Minnesota State University, Mankato
Mankato, MN 56001
(507) 389-1229

Course Title Math 497 Mathematics Capstone Experience (3 Semester Hours)

Description This course is designed to allow undergraduate students an opportunity to integrate their undergraduate mathematics experiences by engaging each student in working on a problem in applied or theoretical mathematics. In doing so, students will see connections between the various topics found in the undergraduate mathematics curriculum. Content will vary by semester. An important component of this course will be the preparation and presentation of a research paper describing the student's progress toward a solution of the problem under consideration. Problems will arise from the course content and materials as presented by the instructor. Because of the breadth of mathematical topics needed for successful completion of the course, students need to have senior standing.

Prerequisites: Two of the following: Math 316 – Intermediate Analysis, Math 345 – Abstract Algebra, or Math 375 – Introduction to Discrete Mathematics and senior standing (or permission of the instructor). Course also can be taken as an independent study with permission of a cooperating faculty member. **F, S**

Goals

1. To engage students in the solution of an applied or theoretical problem in mathematics that enables each student to integrate the knowledge gained in their undergraduate mathematical experience.
2. To communicate mathematics in both written and oral form.

Objectives

1. Students enrolled in the course, in conjunction with the course instructor(s), will select an appropriate problem for a research topic.
2. Students will learn to make mathematical conjectures, develop appropriate solution strategies, and construct valid mathematical arguments in the course of working toward a solution to their selected problem.
3. Students will demonstrate reasonable progress toward a solution to their selected problem.
4. Students will make professional oral and written presentations of their progress toward solving their selected problem. Presentations should include the historical context or origin, and mathematical significance of the problem.
5. Students will engage in a group project activity.
6. Students will use appropriate mathematical language to communicate their results.
7. Students will demonstrate an understanding of written mathematical concepts at an appropriate level.
8. Students will participate in the presentation of the course material.

Evaluation

Suggested methods of evaluation could include, but not be limited to: Class participation, portfolios, article presentations and critique, miscellaneous assignments, individual projects, and a group project.

Possible Course requirements

Assignments, projects, readings, demonstrations, presentations, journals, and portfolio. Student participation in presentation of course material is expected.

Course Resources See attached sample syllabi.

Departmental Resources Needed: May be team taught, or taught by individual instructor depending upon the semester.

Spring 2007

Math 497

Senior Capstone

Math 497

Wissink Hall # 286 (M, W, F: 12:00-12:50 pm)

- **Course Description:** This is a Math/Stat senior capstone course. Topics in this course vary from year to year. This year we will study *Dynamical Systems*. A dynamical system is anything that moves or changes in time. For examples we may list the motion of planets, the weather, stock markets, and chemical reactions. Does it vary continuously in time, or does it change in discrete steps of time? Is it fully deterministic, or does it involve a stochastic process? Can we predict what will happen in the near future for a dynamical system? Will the behavior of the system be robust and stable? These will be some of the principal and motivational questions throughout this course.

This course will be taught by three professors as a team. Professors Namyong Lee, Ernest Boyd and Pavel Kitsul will lead us in the study of continuous dynamical systems, discrete dynamical systems and stochastic processes, respectively, to compare and contrast the mathematical approaches used in each field. Each week the three professors will interact with the students to show the common themes between the different fields. We will introduce concepts and develop the mathematical analysis through applications from the natural sciences, business and economics. We also plan to give a series of short tutorials on using mathematical software as tools for analysis and simulations.

In this course, students are expected to perform a series of weekly assignments and to develop a final project to be presented both orally and written during the final weeks of the semester. The three professors will assist the students throughout the semester to prepare their presentations.

- **Prerequisite:** Calculus (Math 121, 122, 223), Linear Algebra I (Math 247), and Foundations of Mathematics (Math 290). Also recommended Differential Equations (Math 321), Concepts of Probability and Statistics (Math 354), Intermediate Analysis (Math 316)
- **Lecturers:**
 - Dr. Ernest Boyd ernest.boyd@mnsu.edu Office: Wissink Hall # 264 (389-1452)
Course Web Page: <http://krypton.mnsu.edu/~7364eb/Math488/index.html>
Office Hours: M, W, F: 11:00-11:50pm or by appointment
 - Dr. Pavel Kitsul pavel.kitsul@mnsu.edu Office: Wissink Hall # 271 (389-6986)

Course Web Page: <http://krypton.mnsu.edu/~kitsul/capstone>

Office Hours: M, W, F: 11:00-11:50pm or by appointment

o Dr. Namyong Lee nlee@krypton.mnsu.edu Office: Wissink Hall # 245 (389-5096)

Course Web Page: <http://krypton.mnsu.edu/~nlee/capstone>

Office Hours: M, W, F: 11:00-11:50pm or by appointment

- **References:** There is no single textbook. However, to experience research in mathematics, students are expected to read references and articles assigned by the professors. If necessary, class handouts will be distributed throughout the course. Reading assignments and references will be listed on the course web pages.
- **Grading:** A student's grade for this course will be determined by a series of assignments and a final project presented in both written and oral form as follows:
 - o Assignments by each professor: $20\% \times 3 = 60\%$
 - o Final Project's Written Report: 20%
 - o Final Project's Oral Presentation: 20%
- **Computer Labs:** To enhance our learning and problem solving skills, we will use modern technology in our course work. Often we will meet in the ACC computer lab, Wissink Hall #116. If needed, we will use the computer lab in Trafton Hall, TR-E#110. Students will be instructed in mathematical software such as *Mathematica*, and will be expected to use them in their presentations. Students may also seek assistance at any time by visiting our offices or by email.
- **Tentative Schedule:**
 - o Week 1: Introduction to n -dimensional linear systems dependent only on time for continuous, discrete and stochastic variables.
 - o Week 2: Analysis of two-dimensional linear systems for equilibria and stability.
 - o Week 3: Analysis of linear systems governed by discrete or continuous time martingale.
 - o Week 4: Introduction to time-dependent linear systems involving diffusion and Brownian motion in a single space variable.
 - o Week 5: Continuation of the analysis of linear systems.
 - o Week 6: Continuation of the analysis of linear systems.
 - o Week 7: Introduction to two and three-dimensional autonomous nonlinear systems.
 - o Week 8: Analysis of autonomous nonlinear systems in continuous time via two-dimensional Poincaré-Bendixson theory and three-dimensional strange attractors.

- Week 9: Analysis of two-dimensional autonomous nonlinear systems in discrete time via the eigenvalues of the Jacobian matrix and the Lyapunov exponents.
 - Week 10: Simulations of linear and nonlinear stochastic dynamical systems with drift and volatility via Ito's calculus and Markov chains.
 - Week 11: Hopf bifurcation and period doubling.
 - Week 12: Catastrophe theory.
 - Week 13: Summation and where do we go from here?
 - Week 14: Final Project's oral presentations.
 - Week 15: Final Project's oral presentations.
 - Week 16: Final Project's written reports due.
- **Expectations:** Students are encouraged to discuss the topics of this course with each other. However, the assignments should be done individually unless specified otherwise. If a student is having difficulty with an assignment, he/she may ask fellow students for assistance in understanding the assignment, but not for assistance in doing the assignment. Students may always ask the professors for assistance at any time.
 - **Note:** MSU provides students with disabilities reasonable accommodation to participate in educational programs, activities or services. Students with disabilities requiring accommodation to participate in class activities or meet course requirements should first register with the Office of Disability Services, located in 0132 Memorial Library, telephone 389-2825, TDD 711, and then contact the professors as soon as possible.

We hope students will discover and experience the beauty of doing mathematics through this course!

Course Proposal

Title: Math 497 Mathematics Capstone Experience

Topic: Enumerative Combinatorics

Instructor: Dan Singer

Course Description: Enumerative combinatorics is the art of counting. To count discrete objects (permutations, combinations, integer partitions, trees, graphs etc) is to understand their structure down to the last detail. Topics will include elementary counting techniques, bijections, recurrence relations, generating functions, combinatorial species, group actions, and Pólya's method for counting objects with symmetry. Students will use these techniques to make conjectures, prove theorems, read research papers, and communicate mathematical ideas in both written and oral form. They will see connections among the following mathematical disciplines: enumerative combinatorics, abstract algebra, real analysis, and number theory.

Prerequisites: Two of the following: Math 316, Math 345, Math 375.

Instructional Materials: Lecture notes, handouts, research papers. I will not be using a textbook, but will consult the books listed in Appendix A. The research papers available for students to choose from are listed in Appendix B.

Course Format: I will give lectures on counting techniques and work out a number of examples throughout the course. Students will be assigned homework on a weekly basis. At the beginning of the course I will hand out a list of research papers and list of projects. Each student must pick a research paper to read and a project to work on. Both the research paper and the project must be presented to the class in the form of an oral presentation. The project must also be submitted in written form. The research paper must be chosen by week 5 and the research project must be chosen by week 10. Students have the option of working alone or in groups of two on all assignments.

Grading Policy: All assignments are awarded points on a 0 to 100 scale. 90-100 points for an A, 80-89 points for a B, 70-79 points for a C, 60-69 points for a D, 0-59 points for an F. Assignments will consist of weekly homework problems, a research paper to be digested and presented to the class, and a project to be presented to the class and also submitted in written form. The final grade will be calculated as follows: 30% homework score average, 35% research paper, 35% project, letter grade to be awarded according to how this weighted average falls on the grade scale indicated above.

Syllabus:

Weeks 1 and 2: addition principle, multiplication principle, inclusion-exclusion

Weeks 3 and 4: bijections, combinatorial arguments for proving binomial coefficient identities, combinatorial arguments for proving fibonacci number identities

Week 5 and 6: partitions and q -series identities

Week 7: recurrence relations

Week 8: generating functions and the Lagrange inversion formula

Weeks 9 and 10: combinatorial species

Weeks 11 and 12: group actions and Pólya theory

Weeks 13, 14, and 15: Reserved for student presentations of research papers and projects.

Projects for a capstone course on enumerative combinatorics:

1. Find the number of $n \times n$ chess boards in which exactly two squares in every row and column are occupied by pawns and the remaining squares are empty.
2. Find combinatorial proofs of binomial coefficient identities found in Riordan's book – as many as possible.
3. Find q -analogues of binomial coefficient identities found in Riordan's book – as many as possible.
4. (Exercise 79 in Andrews.) Prove that the number of partitions of n in which no part appears exactly once equals the number of partitions into parts not congruent to $\pm 1 \pmod{6}$.
5. (Exercise 102 in Andrews.) show that the generating function for self-conjugate partitions with each part $\leq N$ is

$$\sum_{j=0}^N q^{j^2} \left[\begin{matrix} N \\ j \end{matrix} \right]_{q^2}.$$

6. (Exercise 5.6 in Stanley, Volume II.) Let $\chi(K_{mn}, q)$ denote the chromatic polynomial of the complete bipartite graph K_{mn} . Show that

$$\sum_{m,n \geq 0} \chi(K_{mn}, q) \frac{x^m}{m!} \frac{y^n}{n!} = (e^x + e^y - 1)^q.$$

7. (Exercise 5.7 in Stanley, Volume II.) Using combinatorial species applied to up-down permutations, give a combinatorial proof of the identities

$$1 + \tan^2 x = \sec^2 x$$

and

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}.$$

8. (Exercise 5.9 in Stanley, Volume II.) Let \mathcal{S} be a “structure” that can be put on a finite set S by choosing a partition of S and putting a “connected” structure on each block, so that the exponential formula is applicable. Let $f(n)$ be the number of structures that can be put on an n -element set, and let $F(x) = E_f(x)$, the exponential generating function of f .

(a) Let $g(n)$ be the number of structures that can be put on an n -set so that every connected component has even cardinality. Show that

$$E_g(x) = \sqrt{F(x)F(-x)}.$$

(b) Let $e(n)$ be the number of structures that can be put on an n -set so that the number of connected components is even. Show that

$$E_e(x) = \frac{1}{2} \left(F(x) + \frac{1}{F(x)} \right).$$

9. (Exercise 5.11 in Stanley, Volume II.)

(a) Let $a(n)$ be the number of permutations in S_n which have a square root. Show that

$$\sum_{n \geq 0} a(n) \frac{x^n}{n!} = \left(\frac{1+x}{1-x} \right)^{1/2} \prod_{k \geq 1} \cosh \frac{x^{2k}}{2k}.$$

(b) Deduce from (a) that $a(2n+1) = (2n+1)a(2n)$. Is there a simple combinatorial proof?

10. (Exercise 5.28 in Stanley, Volume II.) Let k be a positive integer. A k -edge colored tree is a tree whose edges are colored from a set of k colors such that any two edges with a common vertex have different colors. Show that the number $T_k(n)$ of k -edge colored trees on the vertex set $[n]$ is given by

$$T_k(n) = k(nk - n)(nk - n - 1) \cdots (nk - 2n + 3) = k(n - 2)! \binom{nk - n}{n - 2}.$$

11. (Exercise 14.45 in Brualdi.) A stained glass window in the form of a 4-by-4 chessboard has 16 squares, each of which is colored red or blue (the colors are transparent and the window can be looked at from the other side). Determine the generating function for the number of different stained glass windows and the total number of stained glass windows.

12. (Exercise 14.49 in Brualdi.) Ten balls are stacked in a triangular array with 1 atop 2 atop 3 atop 4. (Think of billiards.) The triangular array is free to rotate. Find the generating function for the number of nonequivalent colorings with the colors red and blue. Find the generating function if we are also allowed to turn over the array.

Appendix A: Source Materials for Lecture Notes

1. *Integer Partitions* by George E. Andrews and Kimmo Erikson. Cambridge University Press, 2004.

2. *Proofs That Really Count: The Art of Combinatorial Proof* by Arthur T. Benjamin and Jennifer J. Quinn. Dolciani Mathematical Expositions, volume 27. Mathematical Association of America, 2003.

3. *Introductory Combinatorics, Fourth Edition* by Richard A. Brualdi. Prentice Hall, 2004.
4. *Counting: The Art of Enumerative Combinatorics* by George E. Martin. Springer-Verlag, 2001.
5. *An Introduction to Combinatorial Analysis* by J. Riordan. Wiley, 1958.
6. *Enumerative Combinatorics, Volume 1* by Richard P. Stanley. Cambridge University Press, 1997.
7. *Enumerative Combinatorics, Volume 2* by Richard P. Stanley. Cambridge University Press, 1999.
8. *Constructive Combinatorics* by Dennis Stanton and Dennis White. Springer-Verlag, 1986.

Appendix B: Research Papers

1. *n-Color partition theoretic interpretations of some mock theta functions*, A.K. Agarwal, The Electronic Journal of Combinatorics 11 (2004), #N14.
2. *Ordered trees with prescribed root degrees, node degrees, and branch lengths*, E. Deutsch, Discrete Mathematics 282 (2004) 89-94.
3. *The Gaussian coefficients and overpartitions*, William Y.C. Chen, James J.Y. Zhao, Discrete Mathematics 305 (2005), 350-353.
4. *Enumerating symmetric directed convex polyominoes*, Emeric Deutsch, Discrete Mathematics 280 (2004), 225-231.
5. *Odd or even on plane trees*, Sen-Pend Eu, Shu-Chung Liu, Yeong-Nan Yeh, Discrete Mathematics 281 (2004) 189-196.
6. *Combinatorial proofs of identities of Calkin and Hirschhorn*, Hong Feng, Zhizheng Zhang, Discrete Mathematics 277 (2004), 287-294.
7. *A combinatorial proof of the sum of q-cubes*, Kristina C. Garrett, Kristen Hummel, The Electronic Journal of Combinatorics 11 (2004), #R9.
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