

# Test Driving an Active Learning Strategy: The Case of Colored Cards

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Teaching Certificate Program Final Project

## *Context*

Math 201 is the first of two required mathematics content courses for intending elementary teachers. My teaching of this course has been re-worked this semester with the encouragement and support of IPESL funding for critical thinking. One of the results of this IPESL work has been a focus on four main ideas that underlie the mathematical and pedagogical structure of K-12 mathematics curriculum and classroom activities. This paper describes an activity designed to engage students with one of these main ideas, *the models we use to represent numbers*.

There are three major families of models for representing numbers in mathematics: *set models*, *area models* and *linear models*<sup>1</sup>. I introduced these models and their characteristics in lecture early in the semester and used them as ways of describing the diagrams in our textbook, the examples I used in class and the manipulative materials we used occasionally in class. When we represent numbers with sets, we are concerned with the number of objects present and unconcerned with their size, color, orientation or location. The idea that three large apples are the same as three small apples is a *set* idea. With sets, we generally don't cut the objects in the set into smaller pieces for if we cut an object in half, we have two objects. In mathematics this property is referred to with the adjective *discrete*. With area models, we are concerned with the

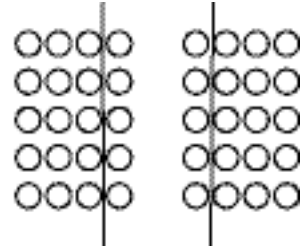
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<sup>1</sup> There are other models that don't fit well into these three categories: *money* is an example, but most of the diagrams and physical materials- called *manipulatives*- that students encounter in K-12 classrooms have characteristics inherent to one of these three categories.

amount of space (area) occupied. Area models are *not* discrete. Pizzas are a common example of an area model; if we cut a pizza into slices, each slice can be expressed as a fraction of the whole pizza. The amount of pizza present does not change according to how many slices we cut the whole into. Furthermore, each slice can be cut into smaller slices. In mathematics, this property is referred to with the adjective *continuous*, which is the opposite of *discrete*. Linear models are also continuous, but the sole attribute to which we pay attention is length.

In the course, I used these three models repeatedly to describe and sort various examples and problems from a variety of sources, including the course textbook. In order to teach critical thinking, it was necessary for students to begin to use these models to organize their own thinking rather than simply to be told the nature of each example. This became particularly clear when an assignment asked students to draw pictures to represent word problems. In reading students' work, it was clear to me that the idea of using a picture to *represent* numbers, their properties and their relationships was challenging for my students. A typical example is when I asked my students to draw a picture to represent the commutative property of addition, which states that for all numbers,  $A + B = B + A$ . Figure 1 shows a very common weak response on the left and a less common strong response on the right. The response on the right is based on a set model. The illustration on the left simply uses alternate symbols for  $A$  and for  $B$ , while the illustration on the right uses the idea that a set model represents numbers by the quantity of objects in the collection and addition by joining two collections.

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**Figure 1:** Two responses to the prompt, *Draw a picture to represent the commutative property of addition*; a weak one on the left and a stronger, set-model response on the right.

I devised an activity to help students see the set, area and linear models underlying mathematical word problems, and then to use these models to draw good pictures for representing these problems.

### *The Activity*

This activity is adapted from an elementary assessment technique (Biggerstaff, 1994). In class, we used a set of six word problems from our textbook, such as the following two (Beckmann, 2005).

*Problem 1.* Suppose that at a certain location, the average daily rainfall in September is  $\frac{5}{8}$  of an inch. Last year, the average daily rainfall at that location in September was only  $\frac{3}{8}$  of an inch. What percent of the average September daily rainfall fell last year in September?

*Problem 2.* Lenny has received 6 boxes of paper, which is 30% of the paper he ordered. How many boxes of paper did Lenny order?

After reading each of the problems, I instructed my students to decide individually whether the context would be best represented by a set model (i.e. we would *count* the objects), an area model (we would pay attention to the amount of *space* taken up) or a linear model (we would pay attention to the *lengths* of the objects in the problem). After a moment to think

silently, the students turned to a neighbor to discuss their choices and the reasons for the choices. The next step was to use colored cards to indicate their choices and to initiate discussion in the large group. We established (arbitrarily) that *yellow* would indicate *set*, *red* would indicate *area* and *blue* would indicate *linear* models. One student in each partnership would, on the count of three, raise colored card of their choice. Students could look around the room and see the extent to which they were in agreement with their classmates and the professor could quickly see this as well.

Problem 1 cited above suggests a linear model; we measure rain with a rain gauge, which generally has a vertical ruler. The length of the filled portion of the gauge is measured. Problem 2 suggests a set model; the boxes are shipped as whole boxes that don't get split up in the shipping process. We are concerned with counting the boxes Lenny received and with counting the boxes he should have received.

We worked through six problems this way in about five minutes. The class struggled with the first few and came to consensus quickly by the end of the activity. After completing the colored card portion of the activity, the final steps were to: (1) make a drawing based on the underlying model for each problem, and (2) solve each problem.

My informal assessment of the activity is that it helped students to visualize and to solve these specific problems, and that it helped students to make progress on the larger goals of the course: to understand and apply the models for representing numbers, and to make rich mathematical drawings that can be used to think and to solve problems, rather than simply to represent known solutions.

*Feedback*

I noticed one pair of students who did not speak much in class was struggling early on with identifying the models in this activity. When they raised the wrong card for the third straight time, one of them (I'll call her *Amber*) started shaking her head and laughing. I made sure to support the class in sharing their thinking publicly and to emphasize the activity as a chance to learn rather than to be evaluated. On a subsequent exam, Amber seemed to have sorted out the model types and I sent her an email letting her know that I had noticed this:

I just wanted to send you a quick note about class recently. I remember you were feeling lost and frustrated when you came to office hours, so I was delighted when your Part I on the exam got a '4'. I hope you felt good about that; it is evidence that you nailed down some important ideas. Further, I sensed that you felt confident in your response of "sets" to the model type for the chip model. Again, this shows progress. Keep up the hard work!

At the end of the semester, Amber may have been thinking about this activity, or others like it when she wrote the following in a course reflection:

I paid attention every day, not only because I wanted to, but also because it was hard not to with all of the in-class activities and interaction with you're [*sic*] neighbor, which I always engaged in also!

### *References*

Beckmann, S. (2005). *Class activities manual to accompany Mathematics for elementary teachers*. Boston: Pearson.

Biggerstaff, M. (1994). Teacher to teacher: Using color to assess mathematical problem-solving. *Arithmetic teacher*, 41, (6), 307—308.