IPESL Project Summary Report

Thinking Critically about Algorithms

Christopher Danielson

July 1, 2007

Purpose

My project focused on Math 201, a required content course for intending elementary teachers. In this course, whole-number and fraction computation algorithms are major foci. My project goal was for students to think critically about these algorithms.

Critical thinking. By thinking critically about algorithms, I mean analyzing student work by answering three questions:

1. What is the student doing?
2. Would this work on a different problem? and
3. Why does this algorithm work or not work in general? (i.e. what representation (e.g. diagrams, counterexamples, etc.) would make the algorithm, or its shortcomings transparent?)

Changes. I have come to believe that there are some basic notions about numbers that need to be emphasized prior to being able to think critically about algorithms for operating on numbers. This became apparent as I worked with students on item 3 above, in which we analyzed various choices about representations. As a result, a major theme developed in the course- I gave students three major representations of number- sets, lines and areas- and had them consider examples of each of these in models from elementary and middle school curricula. Thus the goal of having students think critically about algorithms forced a restructuring of the content of the course. This restructuring required substantial planning effort on the part of the instructor.

Results

Critical thinking about algorithms is not the focus of any textbook of which I am aware. Our current textbook (Beckmann, 2005) has some exercises that address this idea, but it is not a major focus. As a result, I needed to design tasks that required students to do this. Over the course of the semester, students completed five papers in addition to their regular homework exercises and their exams. All of the papers were designed to force students to think in new ways. Two of them were designed to help students to think critically about algorithms: Paper 4 introduced a new whole-number multiplication algorithm, while Paper 5 introduced a new fraction division algorithm. These are attached as Appendices A and B.

At the end of the course, in addition to the usual prompts on the standard course evaluation, I included three of my own questions. One of them asked students to "Comment on the professor's success in getting you to think." No suggestions were made to students about *how* I tried to do this or what feedback would be useful; I simply told students that it was a goal of the course and that I wanted feedback on my success with it.

Ninety-three percent (76 out of 82) of students who completed an evaluation form reported that the course was successful in getting them to think. Two percent (2 out of 82) reported that the course was unsuccessful, while 5% (4 out of 82) did not respond to the prompt. Of those who reported that the course was successful in getting them to think, 16% specifically mentioned the papers as having been a valuable tool for doing so. Recall that these mentions were not prompted, nor were students asked for evidence that they had been made to think.

I was not able to collect the longitudinal evidence I planned to collect in my proposal. Instead, I will report the results of the application of the IPESL critical thinking rubric. At the beginning of the semester, Task 1 was a homework problem from our textbook in which students were asked to comment on a procedure for rounding decimals. Near the end of the semester, Task 2 was an exam problem in which students were asked to choose the written algorithm that most closely matched a student's work with base-10 blocks for whole-number addition. The results from the application of the rubric to the student responses are in Table 1.

Student ID	Task 1 score	Task 2 score
1	4	4
2	2	2
3	4	3
4	2	3
5	3	3
6	2	2
7	3	4

Table 1: IPESL Critical Thinking Rubric scores

The results of this assessment do not show the kind of growth I believe took place in the course. One difficulty with the data is the differing nature of the two items; more carefully chosen items would produce more accurate measurement of student growth. A future version of the course will include a research component that looks carefully at student growth in critical thinking about algorithms.

Issues

The main issues that arose in this IPESL project were related to project evaluation. Because my work this semester involved substantial course redesign, there was little time remaining to plan carefully for meaningful project evaluation. The qualitative evidence I presented above from the course evaluations gives evidence that students thought hard during the semester, and that my course redesign efforts were successful at making this happen, but it does not give evidence of growth in critical thinking- nor could it. I am left with a well-designed course and a need to invest time and effort in evaluating the learning taking place in it. As mentioned above, these issues will be resolved in a future version of the course.

Dissemination

I currently plan to evaluate the Math 201 course in the fall semester and to report the results of the evaluation, together with a description of the course redesign that IPESL supported in the spring, at a Mathematics Education Colloquium and at the annual conference of the Minnesota Council of Teachers of Mathematics in April, 2008.

I have longer-term plans for dissemination as well. In the next five years I hope to write a short, supplementary textbook for use in courses like Math 201 at other institutions- and to use with my own students. In addition, my research agenda for the next five years will focus on the use of diagrams in teaching and learning mathematics. In my IPESL course redesign, I began to work with preservice teachers on the mathematical structure of the diagrams they draw in solving numerical problems; in the fall I have been funded with a Faculty Research Grant to study the diagrams preservice secondary teachers draw with computer software to demonstrate geometric principles. My research on diagrams in mathematics teaching and learning can be expected to produce a steady stream of publications beginning the next 18 months.

References

Beckmann, S. (2005). *Mathematics for elementary teachers.* Boston: Pearson.

Appendix A

Math 201, sections 1 2 and 3
Paper 4: Lattice algorithm for multiplication

Due: Thursday, Apr. 12

Please use this sheet as your cover sheet.

Consider the lattice algorithm for multiplication on the following pages.

1. Practice the algorithm on a variety of problems.
2. Compare and contrast this algorithm with others you know for whole-number multiplication. Your work should include consideration of some of the following ideas from the course:
   - Whether the algorithm works for *all* problems
   - Place value and number relationships
   - Associative and commutative properties of multiplication, and the distributive property of multiplication over addition
   - Relationships to set, area and/or linear models
   - The Lesh Model for mathematical understanding
   - Ability to generalize to other number types

Appendix B

Math 201, sections 1 2 and 3
Paper 5: A New Fraction Division Algorithm?

Due: Tuesday, May 1

Please use this sheet as your cover sheet.
Do not attempt to answer the questions on this paper.

To solve $\frac{3}{4} \div \frac{2}{5}$, Elihu writes:

$$\frac{3}{4} \div \frac{2}{5} \textit{ is the same as } \frac{15}{20} \div \frac{8}{20}\textit{, so the answer to } \frac{3}{4} \div \frac{2}{5} \textit{ is the same as } 15 \div 8.$$

A. Is Elihu's claim that $\frac{3}{4} \div \frac{2}{5}$ is the same as $\frac{15}{20} \div \frac{8}{20}$ correct? Explain.

B. Is his claim that the answer to $\frac{15}{20} \div \frac{8}{20}$ is the same as the answer to $15 \div 8$ correct? Explain.

C. Use Elihu's method to solve 5 more fraction division problems (from class notes, or of your own design). For each problem, draw a picture that helps to justify and illustrate your calculations.